Design Charts for Inhomogeneous Coupled Microstrip Lines

ALAIN E. ROS

Abstract—**Owing to the linear variation of the odd- and even-mode capacitances in terms of the dielectric constant ϵ_r of the substrate, it has been possible to obtain formulas giving all the electrical characteristics of the lines for given values of the ratios, width w and separation s of the lines upon height h of the dielectric substrate. Conversely, by means of charts, it is possible to obtain the geometrical dimensions of the lines, whatever the value of ϵ_r, for the imposed matching impedance and coupling coefficient. The use of these charts is very easy, and the results are the closest to experimental data published to date.**

I. Introduction

In a recent paper [1], we have reported that the capacitance of a single microstrip line is a linear function of the permittivity of the substrate. We examine here the case of two lines of the same width w, separated by a slot of width s; h is the height of the substrate. This structure, parallel coupled microstrip lines, is currently used in MIC technology and has been extensively studied during the past few years. Many theoretical results have already been given [2]–[6].

In this paper, we have compared our results obtained from charts (Fig. 8) to previous theoretical ones, and they have been found to be in close agreement with the published experimental data and always within the error claimed for these data [7].

II. Analysis of Coupled Microstrip Lines

The previous results, obtained for single lines, have also been observed here: in a very large range of values of w/h and s/h ($0.125 \leqslant w/h \leqslant 10$, $0.06 \leqslant s/h \leqslant 8$), the capacitances of the odd and even modes are linear functions of the dielectric constant of the substrate (Figs. 1, 2, and 3). Then, if C_e and C_o are, respectively, the capacitances of the even and odd modes, we can write

$$C_i = p_i(\epsilon_r - 1) + C_{oi}, \qquad i = \begin{cases} e, & \text{for the even mode} \\ o, & \text{for the odd mode.} \end{cases} \tag{1}$$

The C_{oi}'s are the capacitances for the homogeneous coupler (without dielectric substrate) and p_i depends only on w/h and s/h. In fact, it is more convenient to use the capacitances between the conducting lines and the ground planes, Γ_{11} and Γ_{22}, and the mutual capacitance Γ_{12}. These

Manuscript received April 20, 1976; revised July 15, 1977.
The author is with the Faculte des Sciences, Laboratoire D'Electronique, Universite de Nice, 06034 Nice Cedex, France.

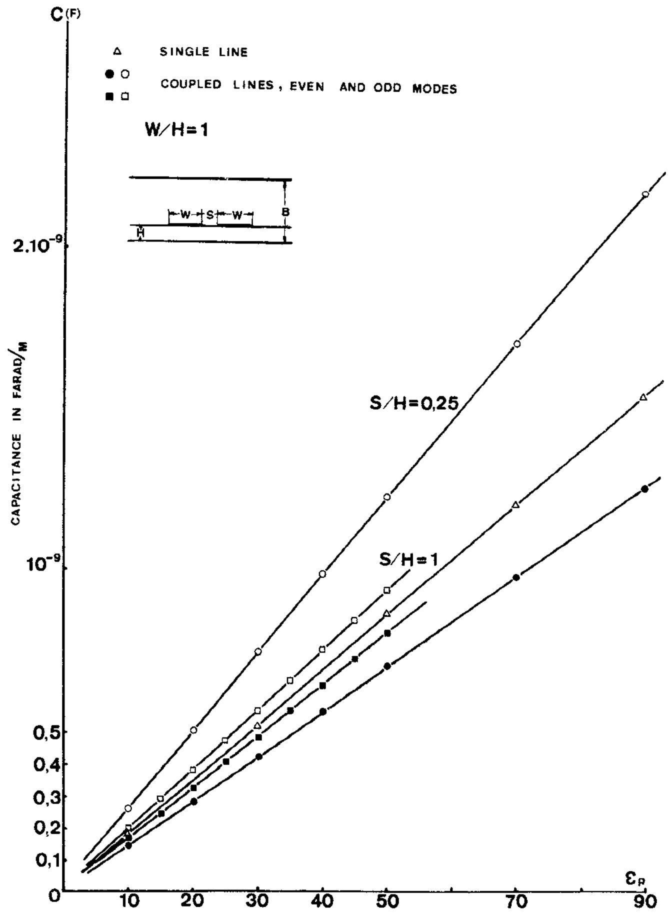

Fig. 1. Odd- and even-mode capacitances in terms of $\epsilon_r \cdot w/h = 1$.

coefficients are independent of the applied potentials ϕ_1 and ϕ_2; this is not the case for C_e and C_o.

The electrical charges on the conductors are

$$Q_1 = \Gamma_{11}(\phi_1 - 0) + \Gamma_{12}(\phi_1 - \phi_2)$$
$$Q_2 = \Gamma_{12}(\phi_2 - \phi_1) + \Gamma_{22}(\phi_2 - 0).$$

For two identical lines, $\Gamma_{11} = \Gamma_{22}$. In the odd-mode case, $\phi_1 = -\phi_2 = V$, so

$$Q_1 = \Gamma_{11}V + 2\Gamma_{12}V$$
$$Q_2 = -2\Gamma_{12}V - \Gamma_{11}V$$
$$Q_1 = -Q_2 = (\Gamma_{11} + 2\Gamma_{12})V$$

and

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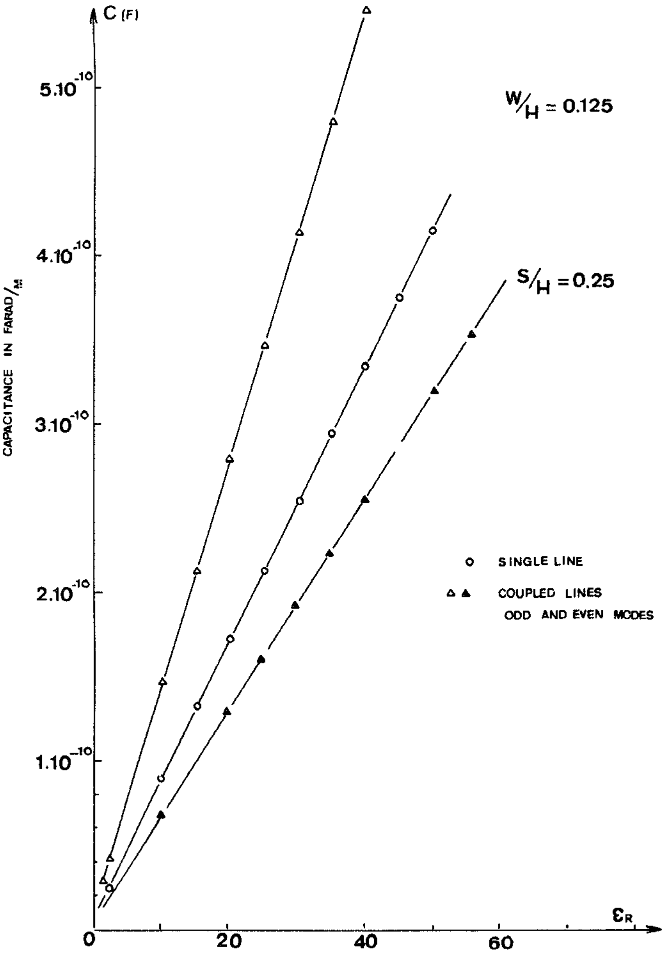

Fig. 2. Odd- and even-mode capacitances in terms of $\epsilon_r \cdot w/h = 0.125$.

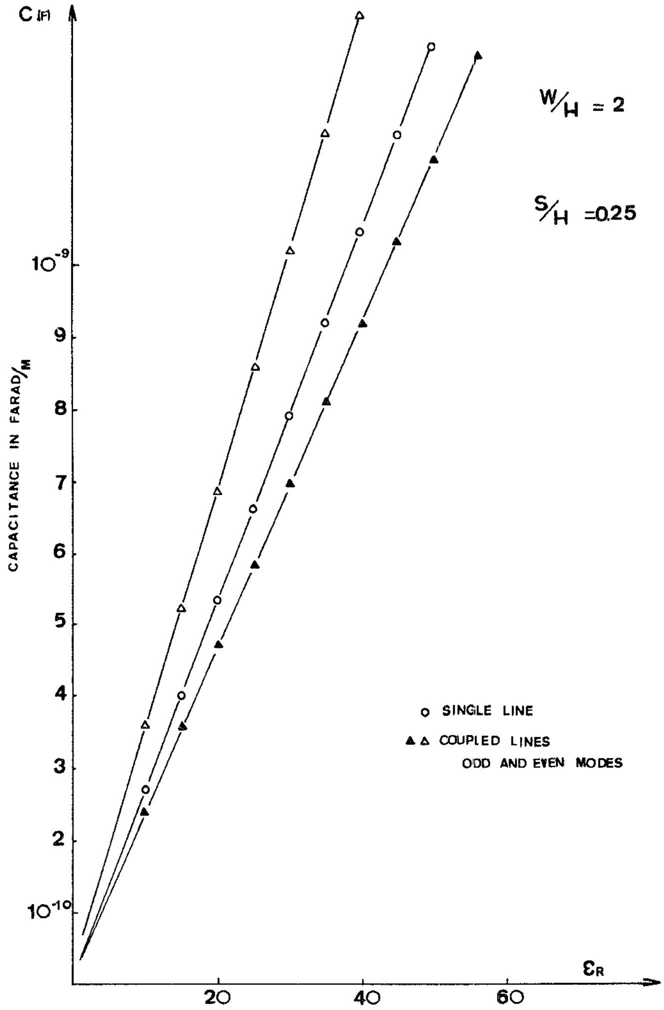

Fig. 3. Odd- and even-mode capacitances in terms of $\epsilon_r \cdot w/h = 2$.

$$C_o = \Gamma_1 + 2\Gamma_{12}.$$

In the even-mode case, $\phi_1 = \phi_2 = V$, so

$$Q_1 = \Gamma_{11} V \quad Q_2 = \Gamma_{11} V \quad \text{and} \quad C_e = \Gamma_{11}.$$

Finally, we have

$$\Gamma_{11} = C_e \qquad \Gamma_{12} = (C_o - C_e)/2. \tag{2}$$

Γ_{11} and Γ_{12} are also linear functions of ϵ_r, so

$$\Gamma_{11} = p_{11}(\epsilon_r - 1) + \Gamma_{110}$$
$$\Gamma_{12} = p_{12}(\epsilon_r - 1) + \Gamma_{120}. \tag{3}$$

In a previous work [8], we described a fast method for computing the electrical characteristics of miscrostrip lines. Using this technique, we have computed more than two hundred pieces of data giving the odd- and even-mode capacitances for these devices. The values of the geometrical dimensions w/h and s/h covered an extended range ($0.125 \leqslant w/h \leqslant 10$, $0.06 \leqslant s/h \leqslant 8$), and the data have been obtained for several values of the relative permittivity ϵ_r, 1, 2.65, and 9.7.

Recognizing the linear variation of C_e and C_o, we have been able to introduce the expressions (3) for Γ_{11} and Γ_{12}. Each set of values of w/h, s/h, C_o, and C_e for $\epsilon_r = 1$ and $\epsilon_r \neq 1$ (9.7 for example), allows us to calculate the corresponding values of Γ_{110}, Γ_{120}, Γ_{11}, and Γ_{12}, and then p_{11} and p_{12}.

p_{11} has been plotted in terms of w/h for different values of s/h (Fig. 4). The asymptotic behavior of these curves shows that we can write, as for the single line

$$p_{11} = p_{11\infty}\frac{w}{h} + a_{011} + a_{111}\left(\frac{w}{h}\right)^{-1} + \cdots + a_{n11}\left(\frac{w}{h}\right)^{-n}.$$

$p_{11\infty}$ is the slope of the oblique asymptotic line; a_{i11} depends only on (s/h).

When s/h tends to infinity, Γ_{11} tends to the capacitance of a single line, so the coefficients a_{i11} tend to those previously obtained for the development of the single microstrip line [1]. Therefore, we can write

$$a_{i11} = a_i + \sum_{j=1}^{n} b_{ij}^{11}\left(\frac{s}{h}\right)^{-j}$$

in matrix form; we can define an $n \times n$ matrix A by

$$P_{11} = p_{11}\infty\frac{w}{h} + SA^{11}W$$

where

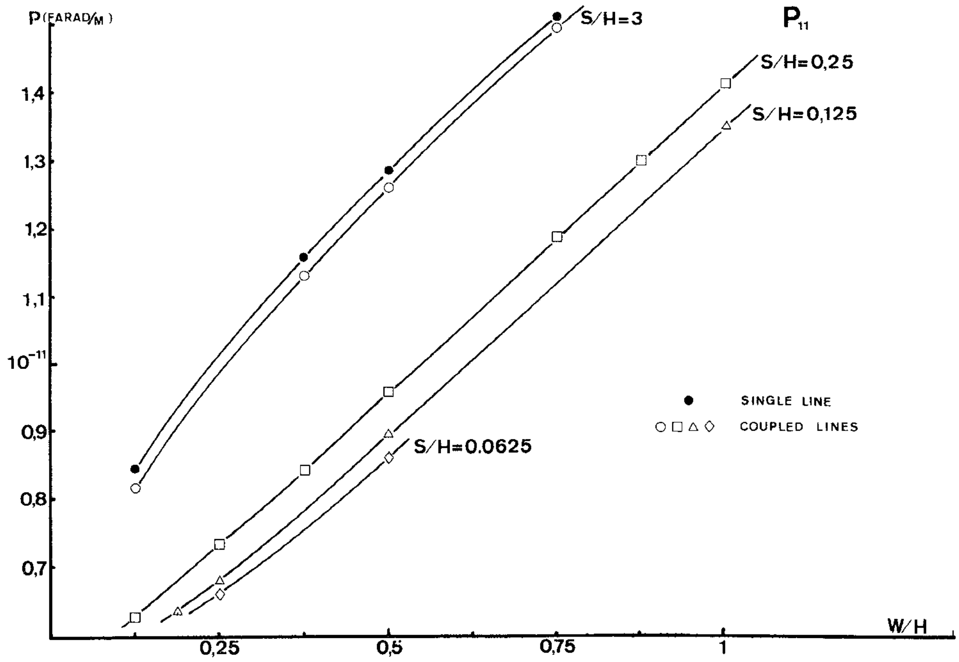

Fig. 4. Slope of the function $\Gamma_{11}=f(\epsilon_r)$ in terms of w/h and s/h.

$$S=\left|\left(\frac{s}{h}\right)^0\left(\frac{s}{h}\right)^{-1}\cdots\left(\frac{s}{h}\right)^{-n}\right| \text{ and } W=\left|\begin{matrix}\left(\frac{w}{h}\right)^0\\ \left(\frac{w}{h}\right)^{-1}\\ \vdots\\ \left(\frac{w}{h}\right)^{-n}\end{matrix}\right|.$$

$$B^{11}=10^{-12}\left|\begin{matrix} 17.9 & -1.35 & 4.02\ 10^{-1} & -4.53\ 10^{-2} & 1.53\ 10^{-3}\\ -5.54 & 1.11 & -6.77\ 10^{-1} & 1.85\ 10^{-1} & -1.40\ 10^{-2}\\ 1.67 & -8.69\ 10^{-1} & 5.59\ 10^{-1} & -1.36\ 10^{-1} & 0.86\ 10^{-3}\\ -1.93\ 10^{-1} & 1.34\ 10^{-1} & -8.64\ 10^{-2} & 2.07\ 10^{-2} & -1.52\ 10^{-3}\\ 6.76\ 10^{-2} & -5.28\ 10^{-2} & 3.38\ 10^{-3} & -8.10\ 10^{-4} & 5.96\ 10^{-5}\end{matrix}\right|.$$

By a least squares method, the elements of A have been calculated, $p_{11\infty}$ and a_i being given by the results for the single line. Finally, if we use fourth-order developments for a_{i11} in terms of $(s/h)^{-1}$, and for p_{11} in terms of $(w/h)^{-1}$, we obtain

$$p_{11\infty}=8.85\,10^{-12}$$

and

$$A^{11}=|a_{ij}|_{\substack{i=0,4\\ j=0,4}}=10^{-12}\left|\begin{matrix} 8.81 & -4.92\ 10^{-1} & 3.53\ 10^{-2} & -1.08\ 10^{-3} & 1.05\ 10^{-5}\\ -1.84 & 2.66\ 10^{-1} & -3.94\ 10^{-1} & 3.24\ 10^{-1} & -5.49\ 10^{-2}\\ 4.18\ 10^{-1} & -1.74\ 10^{-1} & 2.56\ 10^{-1} & -1.77\ 10^{-1} & 2.87\ 10^{-2}\\ -4.11\ 10^{-2} & 2.44\ 10^{-2} & -3.69\ 10^{-2} & 2.46\ 10^{-2} & -3.95\ 10^{-3}\\ 1.32\ 10^{-3} & -9.04\ 10^{-4} & 1.40\ 10^{-3} & -9.24\ 10^{-4} & 1.48\ 10^{-4}\end{matrix}\right|.$$

Γ_{110} has the same behavior as p_{11}. Using the same method, we obtain here

$$\Gamma_{110}=\gamma_{11_\infty}\frac{w}{h}+\sum_{i=0}^{n}\gamma_{i11}\left(\frac{w}{h}\right)^{-i}$$

with

$$\gamma_{i11}=a_{io}+\sum_{j=1}^{n}\delta_{ij}^{11}\left(\frac{s}{h}\right)^{-j}$$

or

$$\Gamma_{110}=\gamma_{11_\infty}\frac{w}{h}+SB^{11}W$$

with

$$\gamma_{11\infty}=8.85\ 10^{-12}$$

and

For the mutual capacitance Γ_{12}, the graphs of p_{12} in terms of w/h for different values of s/h show horizontal asymptotical directions (Fig. 5). This corresponds to the fact that, when w/h tends to infinity, the coupling capacitance depends only on the width of the slot between the two conductors (and on ϵ_r). In addition, when s/h becomes very large, the coupling tends to zero. So, here

$$p_{12}=\sum_{i=0}^{n}a_{i_{12}}\left(\frac{w}{h}\right)^{-i},\qquad a_{i_{12}}=\sum_{j=1}^{n}b_{ij}^{12}\left(\frac{s}{h}\right)^{-j}$$

or

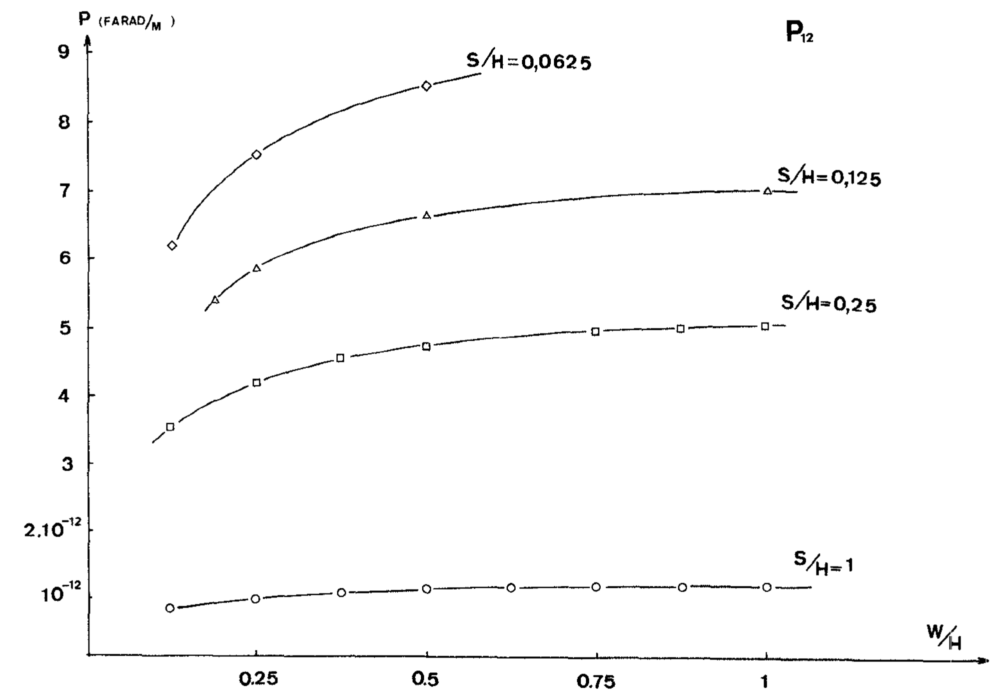

Fig. 5. Slope of the function $\Gamma_{12}=f(\epsilon_r)$ in terms of w/h and s/h.

$$p_{12}=S\cdot A^{12}\cdot W$$

with

$$A^{12}=10^{-12}\begin{vmatrix} 0 & 0 & 0 & 0 & 0 \\ 1.96 & -2.75 & 2.29 & -1.46 & 1.88\ 10^{-1} \\ -1.46\ 10^{-1} & 1.23 & -1.13 & 7.19\ 10^{-1} & -9.28\ 10^{-2} \\ 3.48\ 10^{-3} & -1.51\ 10^{-1} & 1.47\ 10^{-1} & -9.48\ 10^{-2} & 1.23\ 10^{-2} \\ -5.29\ 10^{-4} & 5.26\ 10^{-3} & -5.31\ 10^{-3} & 3.47\ 10^{-3} & -4.53\ 10^{-4} \end{vmatrix}.$$

In the same way, we have

$$\Gamma_{120}=\sum_{i=0}^{n}\gamma_{12}\left(\frac{w}{h}\right)^{-i}, \qquad \gamma_{i_{12}}=\sum_{j=1}^{n}\delta_{ij}^{12}\left(\frac{s}{h}\right)^{-j}$$

or

$$\Gamma_{120}=S\cdot B^{12}\cdot W$$

with

$$B^{12}=10^{-12}\begin{vmatrix} 0 & 0 & 0 & 0 & 0 \\ 8.12 & -5.60 & 2.54 & -4.93\ 10^{-1} & 3.18\ 10^{-2} \\ -1.85 & 2.38 & -1.18 & 2.38\ 10^{-1} & -1.59\ 10^{-2} \\ 1.82\ 10^{-1} & -2.89\ 10^{-1} & -1.48\ 10^{-1} & -3.06\ 10^{-2} & 2.12\ 10^{-3} \\ -5.67\ 10^{-3} & 1.00\ 10^{-2} & -5.23\ 10^{-3} & 1.09\ 10^{-3} & -7.81\ 10^{-5} \end{vmatrix}.$$

Therefore, for given values of w/h, s/h, and ϵ_r, one can easily calculate Γ_{11} and Γ_{12}, that is to say C_e and C_o. By a convenient computer program, or more simply, using a pocket calculator we can determine rapidly the characteristic impedances Z_{oe}, Z_{oo} and the phase velocities v_e, v_o for the two modes directly from the geometrical dimensions of the line since

$$Z_{oi}=\frac{1}{c\sqrt{C_{oi}C_i}} \quad \text{and} \quad v_i=c\sqrt{\frac{C_{oi}}{C_i}}$$

where c is the speed velocity of the light. In the case where v_e and v_o are not too different, we obtain the approximative matching impedance Z_o and the coupling coefficient κ by

$$Z_o=(Z_{oe}\cdot Z_{oo})^{1/2} \text{ and } \kappa_{dB}=20\log\frac{Z_{oe}-Z_{oo}}{Z_{oe}+Z_{oo}}.$$

When the phase velocities are appreciably different, the expressions giving Z_o and κ are more complicated [9], but

TABLE I

		2.65		9.70	
W/H	S/H	V_O (10^8 m/s)	V_E	V_O	V_E
0.5	0.25	2.2085	2.1262	1.2865	1.2040
	0.5	2.2016	2.1122	1.2779	1.1898
1	0.25	2.1934	2.0882	1.2694	1.1674
	0.5	2.1835	2.0852	1.2593	1.1647
	1	2.1671	2.0871	1.2429	1.1664
2	0.5	2.1653	2.0661	1.2412	1.1472
	1	2.1468	2.0701	1.2229	1.1509
	2	2.1244	2.0810	1.2013	1.1608

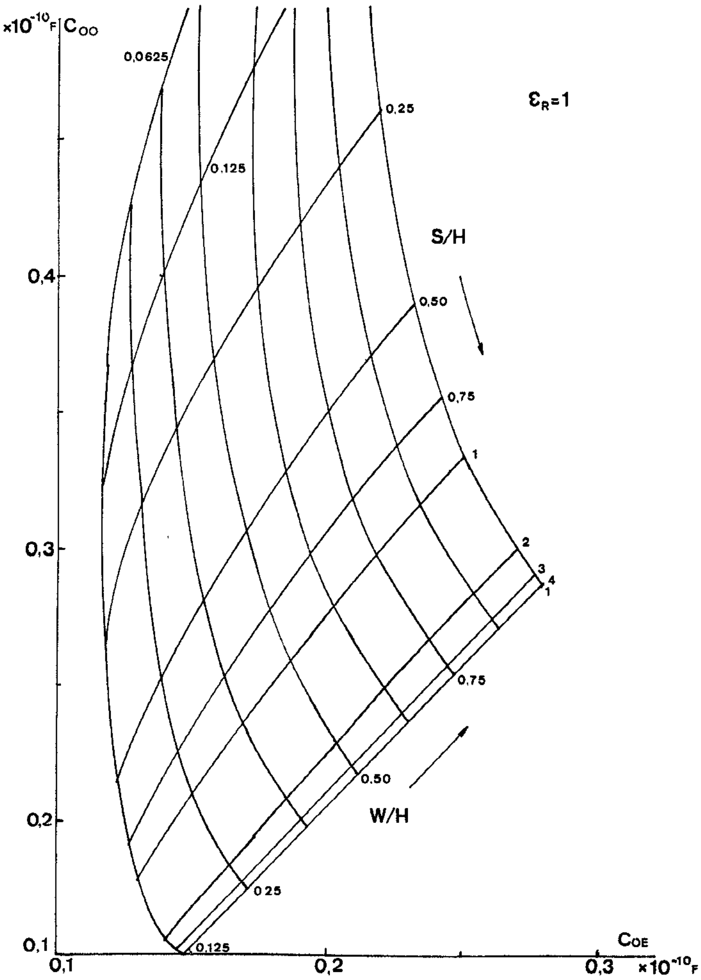

Fig. 6. Odd- and even-mode capacitances for a homogeneous microstrip coupler.

in this case, a computer program will give the accurate analysis of a microstrip coupler.

Table I gives the same couples of values of v_e and v_o for two values of ϵ_r. The gap between the two velocities increases when s/h decreases and when ϵ_r increases; it can reach more than 10 percent. So, for exact analysis (or synthesis) of microstrip couplers, we have to take into account this gap and use a computer program.

But generally, an approximate value of Z_o and κ for given values of w/h and s/h is sufficient (or, of w/h and s/h for given values of Z_o and κ: synthesis). In this case, owing to the linear variation of C_e and C_o in terms of ϵ_r, the use of charts can be very useful.

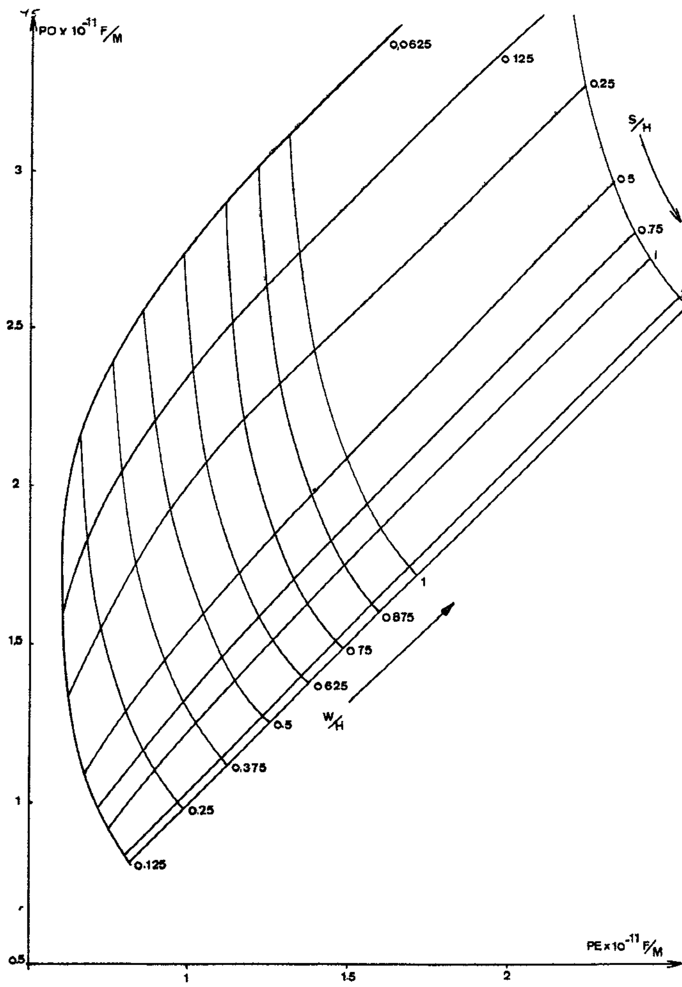

Fig. 7. Slopes of the function $C_i = f(\epsilon_r)$ in terms of w/h and s/h.

Drawing a chart giving C_e and C_o in a homogeneous media (Fig. 6) and another one giving the slopes p_e and p_o of the linear variation of C_e and C_o with ϵ_r (formula 1) (Fig. 7), we can obtain the capacitances of the two modes whatever w/h, s/h, and ϵ_r, and this without any complicated calculation.

III. Synthesis of an Inhomogeneous Microstrip Coupler

The problem is, for example, to determine w/h, s/h giving fixed values of κ and Z_o for a particular value of the permittivity of the substrate.

For an accurate synthesis, taking into account the differences between the phase velocities, it is necessary to use a computer and to proceed by successive approximations. This technique is possible and takes a few seconds on a minicomputer such as the Hewlett–Packard 2100. If we are satisfied with the currently used approximation of the equality of the phase velocities, the same computation can be carried out with a pocket calculator, also by successive approximations. But in this case it will be more convenient to use charts.

Indeed, the more used substrates have generally well-known dielectric constants as for teflon, alumina, quartz,

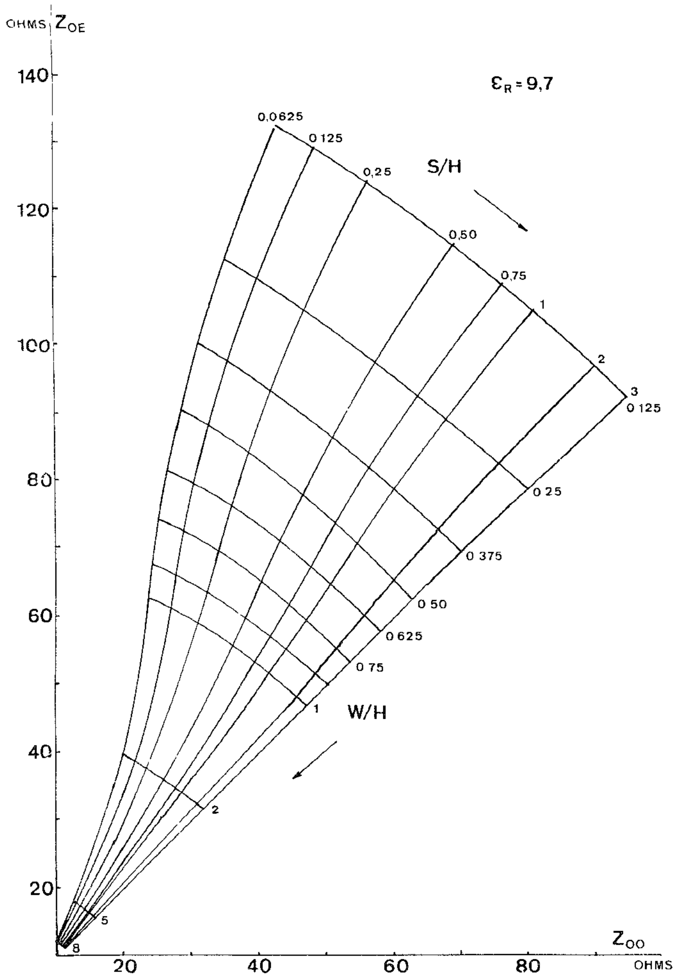

Fig. 8. Odd- and even-mode impedances in terms of w/h and s/h.

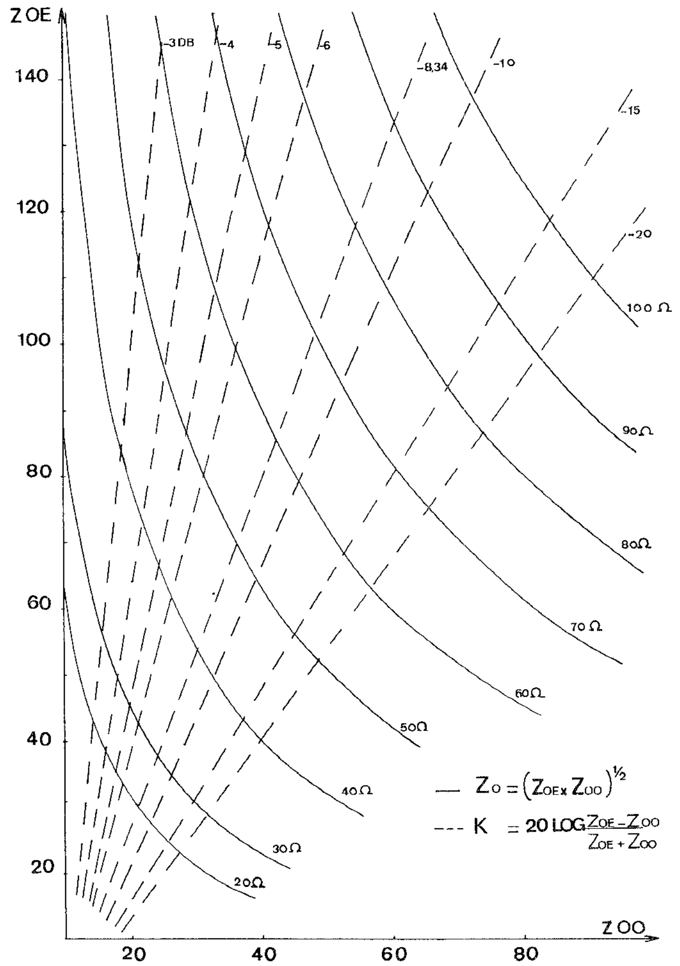

Fig. 9. Matching impedance and coupling coefficient in terms of Z_{oo} and Z_{oe}.

etc. It is then possible to draw the chart giving Z_{oe} and Z_{oo} in terms of w/h and s/h for the desired value of ϵ_r (Fig. 8). This (or these) chart will be able to be used indifferently for analysis of synthesis. In both cases, we have previously drawn (Fig. 9), on a transparency, the curve family corresponding to

$$\kappa = 20 \log \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} = cte, \qquad \text{which are straight lines}$$

and $Z_{oe} \cdot Z_{oo} = Z_o^2 = cte$, which are hyperbolic curves.

By simple superposition, we can determine w/h and s/h corresponding to given values of κ and Z_o, and conversely.

IV. Conclusion

This method, which obliges in some cases to make an interpolation between two curves has, nevertheless, a very good accuracy. In fact, the results obtained in this way are nearer to the experimental data given by Napoli and Hughes [7] than the other theoretical values published to date (Table II). The differences between our theoretical results and the experimental data are always less than 2.7 percent (except for one value: $w/h = 0.368$, $s/h = 0.8$, $Z_{oe} = 86\ \Omega$, for which the discordance reaches 12 percent but is also increased for the other theoretical data: 24 percent for Bryant–Weiss' results, so there is probably a misprint in the paper of Napoli and Hughes). The measurement accuracy being estimated as ± 3 percent, we can say that our method is a very accurate one.

If the use of the formulas giving the developments of p_{11}, Γ_{110}, p_{12}, and Γ_{120} seems to be tedious, the deduced charts lead to data satisfactory for practical work.

TABLE II

W/H	S/H	BRIANT & WEISS (2)		AKHTARZAD (6)		NAPOLI & HUGHES (7)		OUR METHOD		ERROR %	
		ZOE Ω	ZOO Ω	ZOE Ω	ZOO Ω	ZOE Ω	ZOO Ω	ZOE Ω	ZOO Ω	ZOE Ω	ZOO Ω
0 176	0 84	146 92	48 10	128 05	46 84	118	37 5	114 79	37,41	2 7	0 2
0 368	0.12	113 34	41 30	103 51	41 21	86	35	95 88	34 61	11 5	1 1
0 785	0 304	71 81	36 55	69 80	37 75	64	34	64 99	33 30	1 5	2 1
0 863	0.528	64 41	40 12	63 04	41 26	58	38	58 42	36 98	0 7	2 7
0 960	1	56.09	42 77	55 31	43 67	52	40	51 24	40 79	1 5	2
				ϵ_r = 10.4 *							

*The theoretical values given by Bryant and Weiss for $\epsilon_r = 9.0$, and by Akhtarzad *et al.* for $\epsilon_r = 9.6$, have been interpolated and corrected, taking into account the linear variation of C_i and ϵ_r. We have

$$Z_{oi} = \left(1/\left(c\sqrt{C_i C_{oi}}\right)\right) \quad \text{and} \quad C_i = p_i(\epsilon_r - 1) + C_{oi}.$$

So

$$(\Delta Z_{oi}/Z_{oi}) = -\frac{1}{2} \quad (\Delta C_i/C_i) = -\frac{1}{2} p_i(\Delta\epsilon_r/C_i).$$

The continuity and extension of the domain of validity, the very good accuracy, the rapidity, and the convenience of the use of our charts make it a very precious tool for the engineers.

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Calculation of Microstrip Bends and *Y*-Junctions with Arbitrary Angle

REZA MEHRAN

***Abstract*—A method is described for calculating the frequency-dependent scattering parameters of microstrip bends and *Y*-junctions with arbitrary angles. Use is made of a waveguide model and an orthogonal series expansion for the fields around the discontinuity of the bend, so that the excitation and propagation of higher order modes can be considered. The transmission properties of the *Y*-junctions are derived from those of the bends by a symmetry consideration. Numerical results are given for two different substrates and are compared with experimental data. Neglecting radiation effects, they are in good agreement.**

I. Introduction

IN THIS PAPER the transmission and reflection properties of microstrip bends of arbitrary angles are investigated theoretically and experimentally. In addition, the results derived for the bends are employed to calculate the scattering matrix of microstrip *Y*-junctions. Design of *Y*-junctions plays a very important role in fabricating microstrip power dividers; if microstrip bends with small declination are connected in series, it should be possible to realize bandpass or bandstop filters. Up to now a calculation method which yields the frequency-dependent scattering parameters of the above mentioned discontinuities has not been described in the literature.

A waveguide mode, which has successfully been employed in earlier papers on other microstrip discontinuities [1]–[5], also is the basis of the theoretical method used in this paper. By employing a similar mathematical procedure, as it is applied in waveguide theory [11], the discontinuity structure is divided into three subareas, which are partly overlapping; for each of the subregions a complete solution of the wave equation is formulated. The continuity conditions governing the electromagnetic fields are satisfied on the surfaces which are common to two adjacent subregions. A system of equations results from this method, from which the unknown field amplitudes, and subsequently the scattering parameters, can be computed. Representative data achieved from the experimental investigations are compared to the theoretical results.

Manuscript received June 8, 1977; revised October 5, 1977.

The author is with the Lehrstuhl für Allgemeine und theoretische Elektrotechnik, Department of Electrical Engineering, University of Duisburg, Duisburg, Germany.

II. Formulation of the Problem

The waveguide model shown in Fig. 1(b) has been introduced for the microstrip line. It consists of electrical walls at the top and the bottom of the line, and magnetic walls at the sides. The effective width w_{eff} of the model as well as the effective dielectric constant ϵ_{eff} are frequency-dependent model parameters [2], [3]. Making use of this model the properties of the microstrip bend, shown in Fig. 1(c), are calculated. For reasons which will become obvious when calculating the *Y*-junction (Fig. 1(d)), one of the walls may also be chosen to be an electric wall.

The geometrical structure of the bend is subdivided into three regions (Fig. 1(c)), and for each of these subregions

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